Course Title: Analytical Dynamics

Course Code: MATH-405

Course Type: Major Math

Prerequisites: Classical Mechanics

Credit Hours: 3 (3 + 0)

Course Objectives: This course focuses on the principles of Lagrangian and Hamiltonian dynamics, which are two reformulations of classical Newtonian mechanics using a mathematical framework of arbitrary coordinate systems for configuration space and scalar quantities such as kinetic energy and potential energy. This formulation generalizes to modern theories in robotics, aerospace, mechanical engineering, relativity and quantum mechanics. The course aims to achieve the following objectives:

- Understand coordinate transformations based on generalized coordinates, virtual work, D'Alembert's principle, and constrained systems (holonomic and non-holonomic).
- Derive Lagrange's equations of motion and apply Lagrange's theory to analyze holonomic and non-holonomic systems, and energy integrals.
- Explore Hamilton's theory: Calculus of variations, Principle of Least Action, phase space, Hamilton's principle and Hamilton's equations.
- Study canonical transformations, Lagrange and Poisson brackets, conservation laws, Noether's theorem, and its applications in dynamics.
- Apply Hamilton-Jacobi theory to solve problems involving Hamilton's principal and characteristic functions.

Course Contents:

Generalized Coordinates: Generalized coordinates, momenta and forces, classification into holonomic and non-holonomic systems, virtual work, D'Alemberts principle, and the $d\delta$ rule for path variations.

Lagranges Theory of Holonomic Systems: Lagrange equations, generalization of Lagrange equations, quasi-coordinates, Lagrange equations in quasi-coordinates, first integrals of Lagrange equations of motion, energy integral.

Hamiltons Theory: Hamiltons principle, generalized momenta and phase space, Hamiltons equations, Ignorable coordinates, Routhian function, Variational principles and Lagrange's equations, derivation of Lagrange's equations from Hamilton's Principle, derivation of Hamiltons equations from the variational principle.

Lagranges Theory of Non-Holonomic Systems: Lagrange equations for non-holonomic systems with and without Lagrange multipliers, extension of Hamiltons Principle for non-holonomic systems.

Canonical Transformations: The equations of canonical transformations, examples of canonical transformations, the Lagrange and Poisson brackets, infinitesimal canonical transformations and conservation theorems in the Poisson bracket formulation.

Hamilton-Jacobi Theory: The Hamilton-Jacobi equation for Hamiltons principal function, the harmonic oscillator problem as an example of the Hamilton-Jacobi method, the Hamilton-Jacobi equation for Hamiltons characteristic function, separation of variables in the Hamilton-Jacobi equation.

Noether's Theorem and Its Applications: Introduction to Noether's theorem and its application in dynamics, emphasizing the connection between symmetries and conservation laws.

Recommended Books:

1. Aruldhas, G., *Classical Mechanics*, PHI Learning Private Limited, 2009.

2. Fowles, G. R., and Cassiday, G. L., *Analytical Mechanics*, Thomson Brooks/Coley, 7th edition, 2005.

3. Wells, D. A., *Theory and Problems of Lagrangian Dynamics*, McGraw Hill Book Company, 1967.

4. Saletan, E. J., Jose, and J. V., *Classical Dynamics: A Contemporary Approach*, Cambridge University Press, 1998.

5. Spiegel, M. R., *Theoretical Mechanics*, McGraw Hill Book Company, 1980.

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